Effects of the Membrane Potential on the Kinetics of Carrier Mediated Ion Translocation

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The effects of the membrane potential ($\Delta\psi_m$) on the initial rate of ion uptake have been analysed for a simple model of a single site mobile carrier. The initial rate of uptake for an ion i with concentration s_i is (1):

$$v = \frac{V_m \cdot s_i}{K_m + s_i} \qquad (1)$$

V_m and K_m depend on the rate- and binding constants and on the concentrations of competing ions on both sites of the membrane. If the electrical field inside the membrane is constant and the free energy profile within the membrane may be represented by a single symmetrical Eyring barrier, the rate constants for the translocation of a complex k equal:

$$r_{k,I} = r^o_{k,I} \sqrt{y_m}^{z_k} \quad \text{and} \quad r_{k,II} = r^o_{k,II} \sqrt{y_m}^{-z_k} \qquad (2)$$

I and II refer to the inward and outward movement respectively, z_k is the valency of the complex and $y_m = \exp(-\Delta\psi_m \cdot F/RT)$.
The dependence of V_m and K_m on the membrane potential then takes the form:

$$V_m = \frac{a_o + a_1(\sqrt{y_m}) }{b_o + b_1(\sqrt{y_m}) + b_2(\sqrt{y_m})^2} \quad \text{and} \quad K_m = \frac{c_o + c_1(\sqrt{y_m}) + c_2(\sqrt{y_m})^2}{d_o + d_1(\sqrt{y_m}) + d_2(\sqrt{y_m})^2} \qquad (3)$$

The coefficients a_k to d_k depend on the model parameters.
Thus both K_m and V_m turn out to be quite complex functions of $\Delta\psi_m$.
For example, in the case of monovalent cation uptake, a hyperpolarisation may cause either a decrease or an increase of K_m; and V_m may increase, show a maximum or be independent of $\Delta\psi_m$. These effects are due to the electrical potential, which not only changes the rates of the translocation steps, but also effects the distribution of the complexes between the two membrane interfaces.

If an ion hyperpolarizes or depolarizes the membrane, its uptake kinetics may show deviations from simple Michaelis-Menten kinetics. In fact, virtually all types of deviations are possible, depending on the values of the model parameters and on the relation between $\Delta\psi_m$ and the ion concentration. In the same way, competing ions may cause deviations from normal competitive inhibition kinetics.

1. Borst-Pauwels, G.W.F.H. (1974) J. Theor. Biol. 48, 183-195.